

Таблица производных

$(C)' = 0$	$(x^a)' = a \cdot x^{a-1}$	$(x)' = 1$	$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(a^x)' = a^x \cdot \ln a$	$(e^x)' = e^x$
$(\log_a x)' = \frac{1}{x \cdot \ln a}$	$(\ln x)' = \frac{1}{x}$	$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$	$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$	$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$	
$(\operatorname{arctg} x)' = \frac{1}{x^2 + 1}$	$(\operatorname{arcctg} x)' = -\frac{1}{x^2 + 1}$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$	$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$			
$(C \cdot U)' = C \cdot U'$	$(U \pm V)' = U' \pm V'$	$(U \cdot V)' = U' \cdot V + U \cdot V'$	$\left(\frac{U}{V}\right)' = \frac{U' \cdot V - U \cdot V'}{V^2}$			
$(f(U))' = f'(U) \cdot U'$: $(\sin(x^3))' = (\sin(U))' = \cos(U) \cdot U' = \cos(x^3) \cdot (x^3)' = \cos(x^3) \cdot 3x^2$						
$((\sin x)^8)' = (U^8)' = 8 \cdot U^7 \cdot U' = 8 \cdot (\sin x)^7 \cdot (\sin x)' = 8 \cdot \sin^7 x \cdot \cos x$						

Примеры

1. $y = 3x^7 + 5 \operatorname{ctg} x$; $y' = (3x^7 + 5 \operatorname{ctg} x)' = 3 \cdot (x^7)' + 5 \cdot (\operatorname{ctg} x)' = 3 \cdot 7x^{7-1} + 5 \cdot \frac{-1}{\sin^2 x} = 21 \cdot x^6 - \frac{5}{\sin^2 x}$.
2. $y = 5 \sin x - 2e^x + 3x^5$, $y' = 5 \cos x - 2e^x + 15x^4$;
3. $y = 2^x \cdot \sin x$; $y' = (2^x \cdot \sin x)' = (2^x)' \cdot \sin x + 2^x \cdot (\sin x)' = 2^x \cdot \ln 2 \cdot \sin x + 2^x \cdot \cos x$
4. $\left(\frac{\cos x}{\ln x}\right)' = \frac{(\cos x)' \cdot \ln x - \cos x \cdot (\ln x)'}{(\ln x)^2} = \frac{-\sin x \cdot \ln x - \cos x \cdot (1/x)}{(\ln x)^2}$;
5. $(\operatorname{tg}(\ln x))' = \left[(\operatorname{tg}(U))'\right] = \frac{1}{\cos^2(U)} \cdot U' = \frac{1}{\cos^2(\ln x)} \cdot (\ln x)' = \frac{1}{\cos^2(\ln x)} \cdot \frac{1}{x}$.
6. $((4x^3 - 2x + 1)^{10})' = [(U^{10})' = 10 \cdot U^9 \cdot U'] = 10 \cdot (4x^3 - 2x + 1)^9 \cdot (12x^2 - 2)$

Задания для тренировки

Вычислить производные

- 1) $y = 4x^2 + 6x + 3$; 2) $y = 3 \sin x + 2$; 8) $y = 3x^4 + 8x^2 - 7x + 2$; 17) $y = 2 \operatorname{tg} x - 3 \ln x + 2^x$;
- 18) $y = x^2 \cdot \sin x$; 19) $y = e^x \cdot \operatorname{ctg} x$; 20) $y = \sqrt{x} \cdot \log_5 x$; 24) $y = \frac{2x^3}{7x+5}$; 26) $y = \frac{\sin x}{e^x}$;
- 26 a) $y = \frac{5x-3}{4x+1}$; 27) $y = \sin(6x+3)$; 28) $y = \sin 9x$; 29) $y = \sin \sqrt{x}$; 30) $y = \sin(\log_4 x)$;
- 31) $y = \ln(\operatorname{tg} x)$; 32) $y = \ln(x^2 + 3x)$; 33) $y = \ln(2^x)$; 36) $y = (6x+1)^{20}$; 37) $y = (8x^2 + 3)^5$;
- 34) $y = \sin^8 x$; 35) $y = \ln^6 x$; 41) $y = \sin^5 4x$

Ответы к заданиям

- 1) $8x+6$; 2) $3 \cos x$; 8) $12x^3 + 16x - 7$; 17) $\frac{2}{\cos^2 x} - \frac{3}{x} + 2^x \ln 2$; 18) $2x \cdot \sin x + x^2 \cos x$; 19) $e^x \cdot \left(\operatorname{ctg} x - \frac{1}{\sin^2 x}\right)$;
- 20) $\frac{\log_5 x}{2\sqrt{x}} + \frac{\sqrt{x}}{x \ln 5}$; 24) $\frac{28x^3 + 30x^2}{(7x+5)^2}$; 26) $\frac{\cos x \cdot e^x - \sin x \cdot e^x}{e^{2x}}$; 26 a) $\frac{17}{(4x+3)^2}$; 27) $6 \cos(6x+3)$; 28) $9 \cos 9x$;
- 29) $\frac{\cos \sqrt{x}}{2\sqrt{x}}$; 30) $\frac{\cos(\log_4 x)}{x \cdot \ln 4}$; 31) $\frac{1}{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x}$; 32) $\frac{2x+3}{x^2+3x}$; 33) $\ln 2$; 36) $120 \cdot (6x+1)^{19}$;
- 37) $80x \cdot (8x^2 + 3)^4$; 34) $8 \cdot \sin^7 x \cdot \cos x$; 35) $\frac{6 \ln^5 x}{x}$; 41) $20 \sin^4 4x \cdot \cos 4x$;