

Таблица неопределенных интегралов

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C; a \neq -1; \quad \int \frac{dx}{x} = \ln|x| + C; \quad \left(\int x^5 dx = \frac{x^6}{6} + C; \quad \int dx = x + C; \quad \int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C; \right)$$

$$\int a^x dx = \frac{a^x}{\ln a} + C; \quad \int 5^x dx = \frac{5^x}{\ln 5} + C; \quad \int e^x dx = e^x + C; \quad \int \cos x dx = \sin x + C; \quad \int \sin x dx = -\cos x + C;$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C; \quad \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C; \quad \int \frac{dx}{x^2 + 1} = \operatorname{arctg} x + C; \quad \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C;$$

$$\int a f(x) dx = a \int f(x) dx \quad \int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int f(x) dx = F(x) + C \Rightarrow \int f(U) dU = F(U) + C \quad \int f(ax+b) dx = \frac{1}{a} F(ax+b) + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C; \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C; \quad \int \frac{x dx}{x^2 + b} = \frac{1}{2} \ln|x^2 + b| + C; \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C;$$

$$\int U dV = UV - \int V dU \quad \text{или} \quad \int U \cdot V' dx = UV - \int V \cdot U' dx$$

1. Дифференциал

$dy = y' dx$

Примеры

Вычислить dy . 1. $y = 3x^2 - 7x$, $dy = (3x^2 - 7x)' dx = (6x - 7) dx$;

2. $y = \arcsin x$, $dy = (\arcsin x)' dx = \frac{1}{\sqrt{1-x^2}} dx$; 3. $y = \log_5 x$, $dy = (\log_5 x)' dx = \frac{1}{x \cdot \ln 5} dx$

Задано dy . Найти y . 4. $dy = 3x^2 dx$, $y = x^3$; (– это первообразная $3x^2$)

5. $dy = \cos x dx$, $y = \sin x$; 6. $dy = \frac{1}{x} dx$, $y = \ln x$; 7. $dy = \frac{1}{1+x^2} dx$, $y = \operatorname{arctg} x$;

Задания

Вычислить dy :

1) $y = 5x^2 + 3x + 2$; 2) $y = 8x + 2$; 3) $y = x + 9$; 4) $y = \ln x$; 5) $y = e^{2x+3}$; 6) $y = \sin(4x + 7)$;
7) $y = \operatorname{tg}(9x - 1)$; 8) $y = x^3$; 9) $y = \operatorname{arctg} x$; 10) $y = x^4 + 3$; 11) $y = 4^x$; 12) $y = \arcsin(x + 3)$.

Задано dy . Найти y .

13) $dy = e^x dx$; 14) $dy = \sin x dx$; 15) $dy = 4x^3 dx$; 16) $dy = \frac{1}{\cos^2 x} dx$; 17) $dy = \frac{1}{\sqrt{1-x^2}} dx$.

2. Неопределённый интеграл

Примеры

1. $\int \left(\frac{5}{\sqrt{1-x^2}} - \frac{3}{x} + 8 \right) dx = 5 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{1}{x} dx + 8 \int dx = 5 \sin x - 3 \ln|x| + 8x + C$;

2. $\int \left(6^x + 3e^x - \frac{9}{\cos^2 x} \right) dx = \frac{6^x}{\ln 6} + 3e^x - 9 \operatorname{tg} x + C$;

3. $\int \cos(2x + 7) dx = \left[\begin{array}{l} \text{замена } y = 2x + 7, \quad dy = y' dx = 2 dx, \\ \Rightarrow 2 dx = dy, \quad \Rightarrow dx = \frac{1}{2} dy \end{array} \right] = \int \cos y \cdot \frac{1}{2} dy = \frac{1}{2} \int \cos y dy =$
 $= \frac{1}{2} \cos y + C = \frac{1}{2} \cos(2x + 7) + C$

$$4. \int \frac{\ln^4 x}{x} dx = \int \ln^4 x \cdot \frac{1}{x} dx = \int (\ln x)^4 d(\ln x) = [y = \ln x] = \int y^4 dy = \frac{y^5}{5} + C = \frac{\ln^5 x}{5} + C;$$

$$5. \int \frac{x^2 dx}{x^3 + 7} = \left[y = x^3 + 7, dy = 3x^2 dx, \Rightarrow x^2 dx = \frac{1}{3} dy \right] = \frac{1}{3} \int \frac{dy}{y} = \frac{1}{3} \ln|y| + C = \frac{1}{3} \ln|x^3 + 7| + C$$

$$6. \int 4 \sin^3 x \cdot \cos x dx = [y = \sin x, dy = \cos x dx] = \int 4y^3 dy = y^4 + C = \sin^4 x + C$$

$$7. \int x \cos x dx = \left[\begin{array}{l} U = x, \quad dV = \cos x dx = V' dx \\ dU = dx, \quad V = \sin x \end{array} \right] = UV - \int V dU = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$8. \int (9x^2 + 7) \cdot \ln x dx = \left[\begin{array}{l} U = \ln x, \quad dV = (9x^2 + 7) dx = V' dx \\ dU = (\ln x)' dx = \frac{1}{x} dx, \quad V = 9 \cdot \frac{x^3}{3} + 7x = 3x^3 + 7x \end{array} \right] = UV - \int V dU =$$

$$= (\ln x) \cdot (3x^3 + 7x) - \int (3x^3 + 7x) \cdot \frac{1}{x} dx = (3x^3 + 7x) \cdot \ln x - \int (3x^2 + 7) dx = (3x^3 + 7x) \ln x - x^3 - 7x + C.$$

Задания

Вычислить интегралы.

Непосредственно. 1) $\int (6x^2 + 8x^3 - 2) dx$; 2) $\int \sqrt[4]{x} dx$; 3) $\int \left(\frac{4}{1+x^2} - \frac{3}{\sqrt{1-x^2}} \right) dx$; 4) $\int \left(\frac{9}{x} - \frac{1}{\sin^2 x} \right) dx$.

Заменой. 5) $\int \cos(x+9) dx$; 6) $\int e^{2x-5} dx$; 7) $\int \cos(x^2) \cdot 2x dx$; ($y = x^2$) 8) $\int \frac{\arctg^4 x}{1+x^2} dx$; ($y = \arctg x$)

9) $\int \frac{dx}{x+6}$; ($y = x+6$) 10) $\int \frac{dx}{4x+1}$; ($y = 4x+1$) 11) $\int \frac{dx}{\cos^2(2x+7)}$; ($y = 2x+7$)

12) $\int \frac{6x^2}{x^3+8} dx$; ($y = x^3+8$) 13) $\int \sqrt[4]{2x+5} dx$; ($y = 2x+5$) 14) $\int 3\sqrt{x-7} dx$; 15) $\int \sqrt{x^2+4} \cdot 6x dx$;

По частям. 16) $\int (x+3) \cos x dx$; ($U = x+3$) 17) $\int (x-8)e^x dx$; ($U = x-8$) 18) $\int 16 \cdot x^3 \ln x dx$.

Ответы

1. 1) $dy = (10x+3)dx$; 2) $dy = 8 dx$; 3) $dy = dx$; 4) $dy = \frac{1}{x} dx$; 5) $dy = 2e^{2x+3} dx$; 6) $dy = 4 \cos(4x+7) dx$;

7) $dy = \frac{9}{\cos^2(9x-1)} dx$; 8) $dy = 3x^2 dx$; 9) $dy = \frac{1}{1+x^2} dx$; 10) $dy = 4x^3 dx$; 11) $dy = 4^x \ln 4 dx$;

12) $y = \frac{1}{\sqrt{1-(x+3)^2}} dx$; 13) $y = e^x$; 14) $y = -\cos x$; 15) $y = x^4$; 16) $y = \operatorname{tg} x$; 17) $y = \arcsin x$.

2. 1) $2x^3 + 2x^4 - 2x + C$; 2) $0,8 \cdot x^{5/4} + C$; 3) $4 \arctg x - 3 \arcsin x + C$; 4) $9 \ln x - \operatorname{ctg} x + C$; 5) $\sin(x+9) + C$; 6) $0,5 \cdot e^{2x-5} + C$;
7) $\sin(x^2) + C$; 8) $0,2 \cdot \arctg^5 x + C$; 9) $\ln|x+6| + C$; 10) $0,25 \cdot \ln|4x+1| + C$; 11) $0,5 \cdot \operatorname{tg}(2x+7) + C$; 12) $2 \ln|x^3+8| + C$;
13) $0,4 \cdot (2x+5)^{5/4} + C$; 14) $2(x-7)^{3/2} + C$; 15) $2(x^2+4)^{3/2} + C$; 16) $(x+3) \sin x + \cos x + C$; 17) $e^x(x-9) + C$; 18) $x^4(\ln x - 1) + C$.